GPS Errors & Estimating Your Receiver's Accuracy

What's the difference between repeatability and accuracy?

Accuracy

The degree of conformance between the estimated or measured position, time, and/or velocity of a GPS receiver and its true time, position, and/or velocity as compared with a constant standard. Radionavigation system accuracy is usually presented as a statistical measure of system error and is characterized as follows:

- Predictable - The accuracy of a radionavigation system's position solution with respect to the charted solution. Both the position solution and the chart must be based upon the same geodetic datum.
- Repeatable - The accuracy with which a user can return to a position whose coordinates have been measured at a previous time with the same navigation system.
- Relative - The accuracy with which a user can measure position relative to that of another user of the same navigation system at the same time.

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Estimated Position Error (EPE)

EPE (1-sigma) = HDOP * UERE (1-sigma)
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Multiplying the HDOP * UERE * 2 gives EPE (2drms) and is commonly taken as the 95% limit for the magnitude of the horizontal error. The probability of horizontal error is within an ellipse of radius 2 drms ranges between 0.95 and 0.98 depending on the ratio of the ellipse semi-axes. User Equivalent Range Error (UERE) is computed in the tables lower on this page.

Estimate the accuracy of your GPS receiver by following these four steps. A set of measurements is worth a thousand expert opinions! Forget about EPEs! Trust your own plot.

1. Find any convenient unobstructed place.
2. Record the UTM coordinates for that place. Don't throw out ANY data points!
3. Make a graphic plot of Eastings and Northings (pencil and paper works really well for this).
4. Repeat steps 2 and 3 (at different time of day and night, and often)

Notes from Wolfgang Rupprecht:
1. It might induce more folks to actually do the experiment themselves if the lead in paragraph were to be something along the lines of "If things were working well and everything followed the theoretical models then these are the errors one would expect to see..." The difference in my mind is that from a strictly scientific standpoint one really can't assume that any of the information transmit from the satellites is correct or that anything the GPS is calculating is correct. It obviously usually is, but there are low-probability exceptions.
2. The other thing that deserves being stressed is that there are two kinds of "accuracy" numbers being tossed around. One is actual meausred accuracy. This is what the GPS system does (or at least did) during the period of the test. The other are target-spec numbers. These are back-of-the-envelope error budgets that the designers have set aside. They might be overly pessimistic outside numbers, they might be overly optimistic. Only real measurements over extended periods of time can tell.
3. I'd like to see everyone build or buy a gps-to-computer cable and log their own NMEA. There is no substitute for checking the numbers oneself.

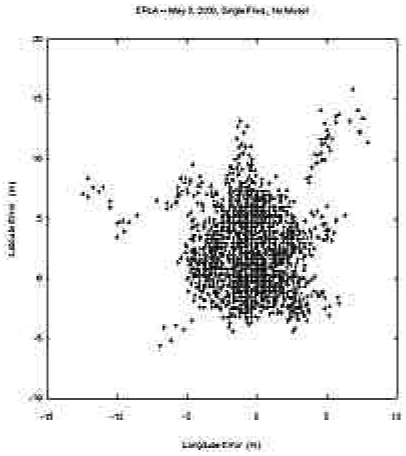
Notes from David L Wilson:
1. Suggest you add something like: The more days and more times per day, the better.
2. Suggest something about the fact that, in general, maximum errors will be larger than those observed. This is due to the fact that large errors are brief and unpredictable. I would definitely include something to this effect.
3. Suggest that if someone is able to log the NMEA data that that is preferred and that what you propose is for those unable to do that or not possesing the software or experience with it to do differently. Maybe you do not have to say that much.
4. Strongly caution that if the person seems to have very different numbers from other that they should be suspicious that they did not collect enough data or that due to probability, they were just lucky or unlucky.

GPS SPS Performance Standard (Oct 2001)
FAA GPS SPS Performance Analysis Report (Jan 2001)

USFS GPS Information Page & Receiver Performance Reports, Including Performance Under Canopy

- Garmin and Magellan Recreational GPS receivers
- Trimble GeoExplorer 3
- Trimble Pathfinder Pro XRS
- Rockwell PLGR 96

PPS Navigation Accuracy Reports



GPS Accuracy Web Pages
from the *National Geodetic Survey (NGS)*

GPS Accuracy Monitor
by *Dennis Milbert*

Garmin 12XL Accuracy Report
by *John Bonde*

Garmins 25LP OEM Receiver Tested
by *Storm Van Leeuwen S.*

Garmin Accuracy Report
by *Wolfgang Rupprecht*

GPS Accuracy Web Pages by *David L. Wilson*, present measured data and mathematical modeling pertaining to GPS accuracy. Visits to a survey marker or short GPS observations will not provide a useful appraisal of GPS accuracy.

Estimation of Trimble Scoutmaster DGPS Performance (1997)
by *Sam Wormley*

The following is adapted from Chapter 11, "**GPS Error Analysis**", pages 478-483, **Global Positioning System: Theory and Applications** by Bradford W. Parkinson, James J. Spilker Jr. Eds.

A. Six Classes of Errors

Ranging errors are grouped into the six following classes:

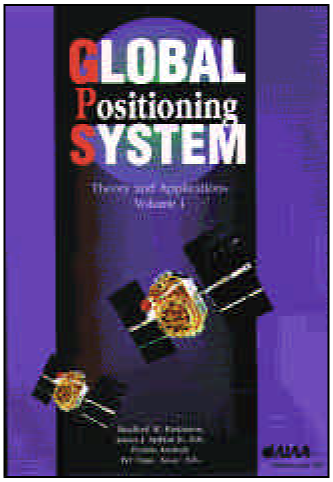
1) ***Ephemeris data***--Errors in the transmitted location of the satellite
2) ***Satellite clock***--Errors in the transmitted clock, including SA
3) ***Ionosphere***--Errors in the corrections of pseudorange caused by ionospheric effects
4) ***Troposphere***--Errors in the corrections of pseudorange caused by tropospheric effects

5) ***Multipath***--Errors caused by reflected signals entering the receiver antenna
6) ***Receiver***--Errors in the receiver's measurement of range caused by thermal noise, software accuracy, and interchannel biases

Each class is briefly discussed in the following sections. Representative values for these errors are used to construct an error table in a later section of this chapter. A more complete discussion of individual error sources can be found in succeeding chapters.

B. Ephemeris Errors

Ephemeris errors result when the GPS message does not transmit the correct satellite location. It is typical that the radial component of this error is the smallest: the tangential and cross-track errors may be larger by an order of magnitude. Fortunately, the larger components do not affect ranging accuracy to the same degree. This can be seen in the fundamental error Eq. (12). The AW represents each satellite position error, but when dot-multiplied by the unit satellite direction vector (in the A matrix), only the projection of satellite positioning error along the line of sight creates a ranging error.



Because satellite errors reflect a position prediction, they tend to grow with time from the last control station upload. It is possible that a portion of the deliberate SA error is added to the ephemeris as well. However, the predictions are long smooth arcs, so all errors in the ephemeris tend to be slowly changing with time. Therefore, their utility in SA is quite limited.

As reported during phase one, (Bowen, 1986) in 1984,[5] *for predictions of up to 24 hours, the rms ranging error attributable to ephemeris was 2.1 m*. These errors were closely correlated with the satellite clock, as we would expect. Note that these errors are the same for both the P- and C/A-codes (see Chapter 16 of this volume for a more detailed discussion of ephemeris and clock errors).

C. Satellite Clock Errors

Fundamental to GPS is the one-way ranging that ultimately depends on satellite clock predictability. These satellite clock errors affect both the C/A- and P-code users in the same way. The error effect can be seen in the fundamental error Eq. (11) as delta-B. This effect is also independent of satellite direction, which is important when the technique of differential corrections is used. All differential stations and users measure an identical satellite clock error.

A major source of apparent clock error is SA, which is varied so as to be unpredictable over periods longer than about 10 minutes. The rms value of SA is typically about 20 m in ranging, but this can change after providing appropriate notice, depending on need. The U.S. Air Force has guaranteed that the twodimensional rms (2 DRMS) positioning error (approximately 90th percentile) will be kept to less than 100 m. This is now a matter of U.S. federal policy and can only be changed by order of the President of the United States. [Note that SA was removed May 2, 2000 @4:05 UTC.]

More interesting is the underlying accuracy of the system with SA off. The ability to predict clock behavior is a measure of clock quality. GPS uses atomic clocks (cesium and rubidium oscillators),' which have stabilities of about I part in 10E13 over a day. If a clock can be predicted to this accuracy, its error in a day (~10E5 s) will be about 10E-8 s or about 3.5 m. The experience reported in 1984 was 4.1 m for 24-hour predictions. *Because the standard deviations of these errors were reported to grow quadratically with time, an average error of 1-2 m for 12-hour* *updates is the normal expectation.*

D. Ionosphere Errors

Because of free electrons in the ionosphere, GPS signals do not travel at the vacuum speed of light as they transit this region. The modulation on the signal is *delayed* in proportion to the number of free electrons encountered and is also (to first order) proportional to the inverse of the carrier frequency squared (1/f squared). The phase of the radio frequency carrier is *advanced* by the same amount because of these effects. Carrier-smoothed receivers should take this into account in the design of their filters. The ionosphere is usually reasonably well-behaved and stable in the temperate zones; near the equator or magnetic poles it can fluctuate considerably. An in-depth discussion of this can be found in Chapter 12, this volume.

All users will correct the raw pseudoranges for the ionospheric delay. The simplest correction will use an internal diurnal model of these delays. The parameters can be updated using information in the GPS communications message (although the accuracy of these updates is not yet clearly established). *The effective accuracy of this modeling is about 2-5 m in ranging for users in the temperate Zones.*

A second technique for *dual-frequency P-code* receivers is to measure the signal at both frequencies and directly solve for the delay. The difference between L1 and L2 arrival times allows a direct algebraic solution. *This dual-frequency technique should provide 1-2 m of ranging accuracy, due to the ionosphere, for a well-calibrated receiver.*

A third technique is to rely on a near real-time update. An example would be the proposed Wide Area Differential GPS system (WADGPS). *This should also produce corrections with accuracies of 1-2 m or better in the temperate zones of the world.*

E. Troposphere Errors

Another deviation from the vacuum speed of light is caused by the troposphere. Variations in temperature, pressure, and humidity all contribute to variations in the speed of light of radio waves. Both the code and carrier will have the same delays. This is described further in the chapter devoted to these effects, Chapter 13 of this volume. *For most users and circumstances, a simple model should be effectively accurate to about 1 m or better.*

F. Multipath Errors

Multipath is the error caused by reflected signals entering the front end of the receiver and masking the real correlation peak. These effects tend to be more pronounced in a static receiver near large reflecting surfaces, where 15 m in or more in ranging error can be found in extreme cases. Monitor or reference stations require special care in siting to avoid unacceptable errors. The first line of defense is to use the combination of antenna cut-off angle and antenna location that minimizes this problem. A second approach is to use so-called "narrow correlatoe, receivers which tend to minimize the impact of multipath on range tracking accuracies. *With proper siting and antenna selection, the net impact to a moving user should be less than 1 m under most circumstances. See Chapter 14 of this volume for further discussion of multipath errors.*

G. Receiver Errors

Initially most GPS commercial receivers were sequential in that one or two tracking channels shared the burden of locking on to four or more satellites. With modem chip technology, it is common to place three or more tracking channels on a single inexpensive chip. As the size and cost have shrunk, techniques have improved and five- or six-channel receivers are common. Most modem receivers use reconstructed carrier to aid the code tracking loops. This produces a precision of better than 0.3 m. Interchannel bias is minimized with digital sampling and all-digital designs.

The limited precision of the receiver software also contributed to errors in earlier designs, which relied on 8-bit microprocessors. With ranges to the satellites of over 20 million meters, a precision of 1:10E10 or better was required. Modem microprocessors now provide such precision along with the co-requisite calculation speeds. *The net result is that the receiver should contribute less than 0.5 ms error in bias and less than 0.2 m in noise. Further information on receiver errors is available in Chapters 3, 7, 8, and 9 of this volume.*

V. Standard Error Tables

These overview discussions on error sources and magnitudes, as well as the effects of satellite geometry, can be summarized with the following error tables. Each error is described as a bias (persistence of minutes or more) and a random effect that is, in effect "white" noise and exhibits little correlation between samples of range. The total error in each category is found as the root sum square (rss) of these two components.

Each *component* of error is assumed to be statistically uncorrelated with all others, so they are combined as an rss as well. The receiver is assumed to filter the measurements so that about 16 samples are effectively averaged reducing the random content by the square root of 16. Of course, averaging cannot improve the bias-type errors.

Finally, each satellite error is assumed to be uncorrelated and of zero mean, so the application of HDOP and VDOP are justified as the last step. Despite these limiting assumptions, the resulting error model has proved to be surprisingly valid. Of course, the assumptions on uncorrelated errors is almost always violated to some degree. For example, if the estimate of zenith ionosphere delay is in error, a proportional error is induced in all measurements through the obliquity calculation. Clearly, such an error would be correlated. These and other correlations have not caused serious problems in the use of this model.

A. Error Table without SA: Normal Operation for C/A Code

Table 2 assumes that SA is not operating. Consequently, the residual satellite clock error, at 2.1 m, is not the dominant error; in fact, the largest error is expected to be the mismodeling of the ionosphere, at 4.0 m. Thus, the worldwide civilian positioning error for GPS is potentially about 10 m (horizontal), as shown in Table 2.

B. Error Table with SA

A second example shows the impact of SA on these errors. Because the deliberately mismodeled clock so dominates the ranging error, all other effects could be safely ignored in the error budget. The results of Table 3 have been repeatedly corroborated by actual measurements. Note that SA is listed as a bias because it cannot be averaged to zero with a 1 s (or less) filter. Selective availability is expected to be zero mean, but only when averaged over many hours or perhaps days. Of course, such averaging is not practical for a dynamic user who only sees the satellite for a portion of the orbit. If differential corrections are used, they will eliminate the SA error entirely (if corrections are passed at a sufficiently high data rate) as discussed in Chapter 21, this volume.

The 41-m horizontal error is a one-sigma result; under the existing agreement between the U.S. Department of Transportation (DOT) and the U.S. Department of Defense (DOD), the 2 DRMS horizontal error is to be less than 100 m. The impact on the vertical error is probably greater, because the VDOP value usually exceeds the HDOP value.

Table 2 Standard error model - L1 C/A (no SA)

	One-sigma error, m			
Error source	Bias	Random	Total	DGPS
Ephemeris data	2.1	0.0	2.1	0.0
Satellite clock	2.0	0.7	2.1	0.0
Ionosphere	4.0	0.5	4.0	0.4
Troposphere	0.5	0.5	0.7	0.2
Multipath	1.0	1.0	1.4	1.4
Receiver measurement	0.5	0.2	0.5	0.5
User equivalent range error (UERE), rms*	5.1	1.4	5.3	1.6
Filtered UERE, rms	5.1	0.4	5.1	1.5
Vertical one-sigma errors--VDOP= 2.5			**12.8**	**3.9**
Horizontal one-sigma errors--HDOP= 2.0			**10.2**	**3.1**

*This is the statistical ranging error (one-sigma) that represents the total of all contributing sources. The dominant error is usually the ionosphere. A horizontal error of 10 m (one-sigma) is the expected performance for the temperate latitudes using civilian (C/A-code) receivers.

Table 3 Standard error model - L1 C/A (with SA)

	One-sigma error, m			
Error source	Bias	Random	Total	DGPS
Ephemeris data	2.1	0.0	2.1	0.0
Satellite clock (dither)	20.0	0.7	20.0	0.0
Ionosphere	4.0	0.5	4.0	0.4
Troposphere	0.5	0.5	0.7	0.2
Multipath	1.0	1.0	1.4	1.4
Receiver measurement	0.5	0.2	0.5	0.5
User equivalent range error (UERE), rms*	20.5	1.4	20.6	1.6
Filtered UERE, rms	20.5	0.4	20.5	1.5
Vertical one-sigma errors--VDOP= 2.5			**51.4**	**3.9**
Horizontal one-sigma errors--HDOP= 2.0			**41.1**	**3.1**

C. Error Table for Precise Positioning Service (PPS Dual-Frequency P/Y Code)

The errors for dual-frequency PN code are similar to those above except that SA errors are eliminated because the authorized user can decode the magnitude as part of a classified message. An expected horizontal error is less than 10 m. The ionosphere error is reduced to 1-m bias and about 0.7 m of noise by the dual-frequency measurement. The dominant sources are the satellite ephemeris and clocks. This is illustrated in Table 4.

Table 4 Precise error model, dual-frequency, P(Y) code

	One-sigma error, m			
Error source	Bias	Random	Total	DGPS
Ephemeris data	2.1	0.0	2.1	0.0
Satellite clock	2.0	0.7	2.1	0.0
Ionosphere	1.0	0.5	1.2	0.1
Troposphere	0.5	0.5	0.7	0.1
Multipath	1.0	1.0	1.4	1.4
Receiver measurement	0.5	0.2	0.5	0.5
User equivalent range error (UERE), rms*	3.3	1.5	3.6	1.5
Filtered UERE, rms	3.3	0.4	3.3	1.4
Vertical one-sigma errors--VDOP= 2.5			**8.3**	**3.7**
Horizontal one-sigma errors--HDOP= 2.0			**6.6**	**3.0**

VI. Summary

Excluding the deliberate degradation of SA, the dominant error source for satellite ranging with single frequency receivers is usually the ionosphere. It is on the order of four meters, depending on the quality of the single-frequency model. For dual-frequency (P/Y-code) receivers (which eliminate SA) the Standard Error Model of Table I has one principal change (in addition to the elimination of the SA error). The ionospheric error is reduced from four meters to about one meter.

Greater variations in the errors are due to geometry, which are quantified as dilutions of precision or DOPs. While geometric dilutions of 2.5 are about the worldwide average, this factor can range up to 10 or more with poor satellite geometry. Reduced satellite availability (and consequent increases in DOP) could be caused by satellite outages, local teff ain masking, or user antenna tilting (for example due to aircraft banking). Typical normal accuracy (one-sigma) for welldesigned civil equipment under nominal operating conditions with SA off should be about 10 m horizontal and 13 m vertical.

References

Martin, E. H., "GPS User Equipment Error Models," *Global Positioning System Papers*, Vol. I, Institute of Navigation, Washington, DC, 1980, pp. 109-118.

Milliken, R. J., and Zollar, C. J., "Principle of Operation of NAVSTAR and System Characteristics," Global Positioning System Papers, Vol. 1, Institute of Navigation, Washington, DC, 1980, pp. 3-14.

Copps, E. M., "An Aspect of the Role of the Clock in a GPS Receiver," *Global Positioning System Papers*, Vol. 111, Institute of Navigation, Washington, DC, 1986.

Massat, P., and Rudnick, K., "Geometric Formulas for Dilution of Precision Calculations," *Navigation*, Vol. 37, No.

4, 1990-1991.

Bowen, R., et al., "GPS Control System Accuracies," *Global Positioning System Papers*, Vol. III, Institute of Navigation, Washington, DC, 1986, pp. 241-247.

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